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Inventory Model of Supply Chain Management 3-Echelon Multi-Tiers

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***Abstract*– The SCM model of 3-echelon is used to optimize the expenditure by minimizing the inventory use that can generally increase the additional expenditure in the system. This paper develops an SCM model of 3-echelon multi-tiers that is comprised suppliers *s*, producers *p*, and retails *r*. The model was developed using deterioration rate assumption of raw material and finished product following Weibull distribution rate. The research use Taylor's expansion method to determine the numeric solution of the model that had been developed. The result indicates that the developed model can be used to optimize the total expenditure in the system of SCM 3-echelon multi-tiers.**

***Keywords*– Three echelon supply chain, deteriorating inventory, Weibull deteriorating, supply chain multi-tiers**

I. Introduction

The highly competitive business environments of today force companies choose good decisions in order to survive [9]. One of these critical decisions may involve integrating the supply chain in which the companies participate to enable the ability to make business decisions jointly [2, 8, 17]. It is important to note that independent decisions benefit only one party of the supply chain [10]. Therefore, the success of a company depends basically on its ability to align all supply chain parties seamlessly [11, 13].

Basically, the costumers expect to be able to obtain a product that has benefits with an acceptable price in a fixed time [14, 10, 18]. In order to achieve these, the company must optimally use their whole ability to satisfy costumer desires. It certainly raises the cost of the different consequences in each company. To be able to offer attractive products at competitive prices, each company would have to reduce all costs without reducing the quality of the products [8, 10, 17].

The company must do efficiency in running its production system in order to create products at competitive prices without reducing the quality of the products. This efficiency can be done in various ways, such as scheduling optimal production process, efficiently use of human resources and inventory management to reduce the inventory cost [11]. Inventories must be managed effectively as a temporary storage of goods to meet all order without any shortage. On the other hand, the inventory system must be efficient because every item that is stored will cause the cost of inventory [8]. Product supply shortages will result in lost sales, while excess inventory products will result in a stacked of products and increased inventory costs [8, 17]. Recently, the inventory management manufacturing companies is often done with the involvement of outside parties that are directly related to the inventory system. Cooperation with external parties to ensure the availability of the required items in the right amount, fixed timing, and the right place is known as supply chain management [16, 19].

For a vertically integrated supply chain owned partially or jointly by the same company, such coordinated production–shipment policy provides valuable insights and optimal decisions that lead to global optimization [20]. On the other hand, when individual entities are owned separately, such policy may not beneficial for all the parties equally as some may encounter an increase in their costs and hence become less eager to depart from their locally optimized policies [8]. In such situations, sharing those benefits resulting from the coordinated approach becomes a major issue. Most of the work related to joint economic lot size production (JELP) has been conducted in the context of a two layers supply chain consisting of a single vendor and a single buyer [21]. Coordinating orders in a two-level (vendor–buyer) supply chain has been addressed in [4]. Meanwhile, [2] observed an integrated production inventory marketing model to determine economic production quantity and economic order quantity for raw materials in a multi-echelon production system.

II. Supply Chain Management

A supply chain is a set of facilities, suppliers, costumers, products, and methods of controlling inventory, purchasing and distribution [7]. The objective is to provide customers with products they want in a timely way as profitable as possible. The main objective is to enhance the operational efficiency, profitability, and competitive position of a firm and its supplier chain partners [6]. Each independent entity of supply chain has inherent objective function to maximize in business transaction for profit maximization.

There are three things that must be managed in the supply chain as follows [3].

1. The flow of goods from upstream to downstream.
2. The flow of cash from downstream to upstream.

3. The information flow can occur from upstream to downstream, or conversely.

In simple terms, a supply chain structure model can be described in Fig. 1 [1].

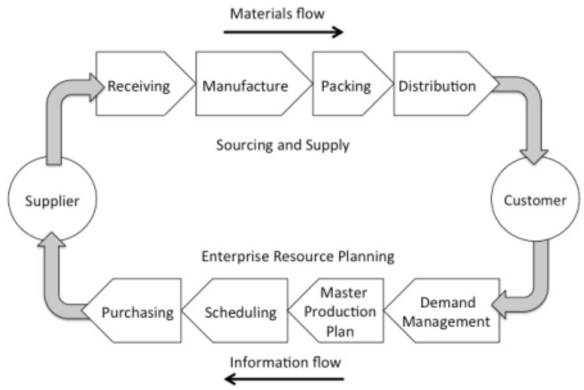

Fig. 1. Flow of materials and information

Generally, the types of supply chain management can be distinguished based on the entities that involved [10, 12]. This paper limits three involved entities, i.e., supplier, manufacturer, and retailer [17, 15, 19]. This kind of SCM is known as 3-Echelon Supply Chain Management.

The supply chain management 3-echelon itself has a various types depending on the amount of each of the entities involved. For example, Singh et. *al* [8] discussed a supply chain management 3-echelon that consisting of one supplier, a manufacturer, and many retailers (multi buyers). In simple terms, connection between the entities can be seen in Fig. 2.

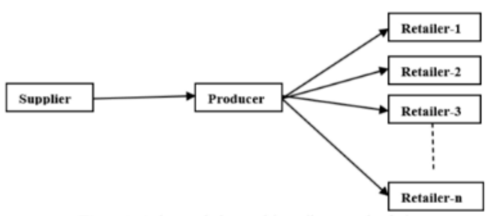

Fig. 2. SCM 3-Echelon Multiple-Buyers

Meanwhile, description of a complex supply chain management proposed by [5] is given in Fig. 3.

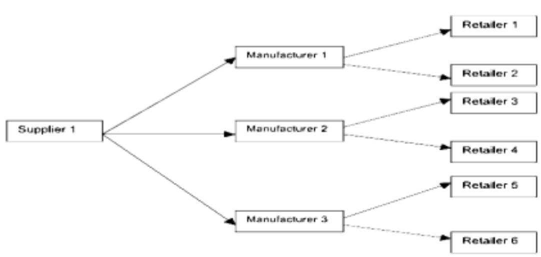

Fig. 3. SCM 3-Echelon Complex

As noted earlier, the focus of this study is a Supply Chain that contains of 3 management entities (termed as echelon) that are supplier of raw materials, manufacturer and retailers [8]. These assumptions below are used in order to develop the model:

1. This SCM consist of single supplier, single manufacturer, and many retailers
2. Deterioration occurs in this process and follows a two-parameter Weibull distribution. The deteriorated units are not replaced.
3. Demand rate and production rate is variable. Production rate is greater than demand rate.
4. Partial backlogging is allowed for buyers only. The partial backlogging is replenished in the next delivery.
5. Multiple deliveries per order are considered.

By using algebra manipulation and Weibull distribution, the model with demand rate and production rate is deterministic and constant [8], was obtained as follows.

$$TUC\ (k, T_2, T_4) = TUC_s + TUC_p + TUC_b$$

The constraints of the objective function are minimized depends on the following parameters.

T_1 = production storage period,
T_2 = production distribution period,
T_3 = period of stock delivered,
T_4 = period of shortage is backlogged and lost sales, and
k = number of deliveries.

III. Inventory Model of SCM 3-Echelon Multi-tiers

The model that we constructed can be seen as the following figure:

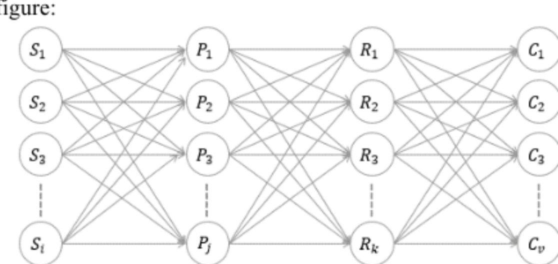

Fig. 4. The SCM 3-Echelon Multi-tiers

Assumed S is supplier, P is the manufacturer, and R is retailers, then we can construct an inventory model of Supply Chain Management with s suppliers, p manufacturers, and r retails. The numeric solutions can be found using Taylor series with $\beta, b, h, r < 0$.

Inventory model of supplier affected by the rate of production and the rate of deterioration of the raw material that assumed to follow Weibull distribution. The inventory level of raw materials from the suppliers can be formulated as given in equation (1).

$$\frac{dI_{w_s}(t)}{dt} = -\sum_{s=1}^{u} P_{s,p} - \alpha\beta t^{\alpha-1} I_{w_t}(t) \tag{1}$$

Hence,

$$I_{w_s}(t) = \sum_{p=1}^{m} P_{s,p}\, e^{-\beta t^{\alpha}} \int_{t}^{T_1} e^{\beta t^{\alpha}} dt \tag{2}$$

By using boundary conditions when $t = 0$, then the maximum inventory level of raw materials can be obtained from the following equation (3).

$$Q_w \approx \sum_{p=1}^{m} P_{s,p} \left(T_1 + \frac{\beta T_1^{\alpha+1}}{\alpha+1} \right) \tag{3}$$

The maximum total cost incurred by the supplier S can be obtained using the following steps.

1. Setup Cost, $S_{w_s} = C_{1w}$.
2. Inventory holding cost that occurs during period T_1,

   $$HD_{w_s} = C_{hw} \int_{0}^{T_1} I_{w_s}(t)\, e^{-rt} dt$$

   $$HD_{w_s} \approx C_{hw} \sum_{p=1}^{N} P_{s,p} \left[\frac{1}{2} T_1^2 - \frac{\beta T1^{\alpha+2}}{(\alpha+1)(\alpha+2)} - \frac{1}{6} r T_1^3 + \frac{\beta T_1^{\alpha+2}}{\alpha+2} \right]$$

3. Item cost that includes loss due to the deterioration as well as the cost the item sold.

   $$IT_{w_s} = C_w Q_w$$

   $$IT_{w_s} \approx C_w \sum_{j=1}^{n} P_{i,j} \left(T_1 + \frac{\beta T_1^{\alpha+1}}{\alpha+1} \right)$$

Therefore, the total cost of raw materials from suppliers is,

$$TUC = S_w + HD_w + IT_w$$

Inventory model of manufacturer p is divided into two periods, i.e., the production period and non-production period. In the production period of the manufacturer, the inventory is affected by the rate of production, the rate of demand from retail r, and the rate of deterioration of the finished product that assumed to follow Weibull distribution. Therefore, the inventory level from manufacturers during the production period can be formulated as in equation (4) and (5).

$$\frac{dI_{p1}(t)}{dt} = P_{p,r} - \sum_{r=1}^{u} D_{p,r} - abt^{a-1} I_{p1}(t) \tag{4}$$

yields,

$$I_{p1}(t) = \left(P_{p,r} - \sum_{r=1}^{u} D_{p,r} \right) e^{-bt^{a}} \int_{t}^{T_1} e^{bt^{a}} dt \tag{5}$$

By using boundary condition when $t = T_1$, where the inventory level of manufacturer during production period reach maximum level can be further obtained using equation (6).

$$MI_p \approx \left(P_{p,r} - \sum_{r=1}^{u} D_{p,r} \right) \left(T_1 - \frac{abT_1^{a+1}}{a+1} \right) \tag{6}$$

During the non-production period, inventory level affected by the rate of demand from retail and the rate of deterioration of the finished product manufacturers that assumed to follow Weibull distribution. Thus, the inventory level for manufacturers during the period of non-production can be formulated the following equations (7) and (8).

$$\frac{dI_{p2}(t)}{dt} = -\sum_{r=1}^{u} D_{r_{p,r}} - abt^{a-1} I_{p2}(t) \tag{7}$$

yields,

$$I_{p2}(t) = \sum_{r=1}^{u} D_{p,r} e^{-bt^{a}} \int_{t}^{T_2} e^{bt^{a}} dt \tag{8}$$

By using the boundary condition when $t = 0$, where the level inventory of producer during the non-production period reached a maximum level can be obtained using equation (9).

$$MI_p \approx \sum_{r=1}^{u} D_{p,r} \left(T_2 + \frac{bT_2^{a+1}}{a+1} \right) \tag{9}$$

The total cost of the manufacturers can be obtained as follows.

1. Setup cost is $S_{p_p} = C_{1_p}$.
2. Ordering Cost is $OR_{p_s} = \sum_{s=1}^{n} C_{2p_s}$.
3. The inventory holding cost occurred during period T_1 and T_2, therefore the total cost of inventory holding is,

   $$HD_p = C_{hp} \left[\int_{0}^{T_1} I_{p1}(t) e^{-rt} dt + I_{p_2}(t) e^{-r(t+T_1)} dt \right]$$

   $$HD_p \approx C_{hp} \left[(P_{s,t} - D_{p,r}) \left(\frac{1}{2} T_1^2 - \frac{bT1^{a+2}}{(a+1)(\alpha+2)} - \frac{1}{6} r T_1^2 + \frac{bT_1^{a+2}}{\alpha+2} \right) \right]$$

   $$- \left[D_{p,r} \left(\frac{1}{2} T_2^2 - \frac{1}{6} r T_2^2 + \frac{1}{2} r T_1 T_2^2 - \frac{bT_2^{a+2}}{(a+1)(a+2)} + \frac{bT_2^{a+2}}{\alpha+2} \right) \right]$$

4. Item cost is,

   $$IT_p = C_p\, Q_p$$

   $$IT_p = C_p \left(\left(P_{p,r} - \sum_{r=1}^{u} D_{p,r} \right) \left(T_1 - \frac{abT_1^{a+1}}{a+1} \right) \right)$$

Thus the total cost of the manufactures is,

$$TUC_p = S_{p_p} + OR_{p_s} + HD_p + IT_p$$

Inventory Model retail is divided into two periods: In the period of stocking, the finished products were distributed to retail are not distributed directly to consumers but is temporarily stored in storage prior to distribution to retail consumers. Thus the change in the retail inventory stocking period is affected by the rate of demand from consumers and the rate of deterioration of the finished product. Therefore, the inventory level retail stocking period can be formulated as the following equation (10).

$$\frac{dI_{r1}(t)}{dt} = -\sum_{k=1}^{v} D_{c_{rk}} - ght^{g-1} I_{r1}(t) \qquad (10)$$

yields,

$$I_{r3}(t) = -\sum_{k=1}^{v} D_{c_{rk}}\, e^{-ht^{g}} \int_{0}^{t} e^{ht^{g}}\, dt \qquad (11)$$

By using the boundary condition $t = 0$, where retail inventory levels during the period of distribution reaches the maximum level, then it can be obtained

$$MI_{r_3} \approx \sum_{k=1}^{v} D_{c_{rk}} \left[T_3 + \frac{1}{g+1} hT_3^{g+1}\right] \qquad (12)$$

During the period of the fulfilment of goods, retailers distribute products to replenish the demand for goods to consumers who previously met most recently in the delivery period. Thus the inventory level of retailer inventory during the period of the fulfilment is not affected by the deterioration and it can be formulated in equation (13) and (14) as follows.

$$\frac{dI_{r4}(t)}{dt} = -B \sum_{c=1}^{v} D_c \qquad (13)$$

yields,

$$I_{r4}(T_4) = -B \sum_{k=1}^{v} D_{c_{rk}}\, t \qquad (14)$$

By using boundary condition $t = T_4$, where the inventory level of during replenish period can be obtained by equation (15).

$$MI_{r_4} = BT_4 \sum_{k=1}^{v} D_{c_{rk}} \qquad (15)$$

The total cost of the retailer can be obtained as follows.

1. The ordering cost:

$$OR_r = k \sum_{i=1}^{n} C_{1r}^{i}$$

2. Total cost of inventory holding cost that occurs during T_2 is,

$$HD_r = k \sum_{i=1}^{n} C_{hb}^{i} \int_{0}^{T_2} I_{r3}(t)\, e^{-rt} dt$$
$$\approx k \sum_{i=1}^{n} C_{rb}^{i} \left(\sum_{k=1}^{v} D_{c_{rk}}\right)\left(\frac{1}{2}T_2^2 - \frac{\beta T_2^{g+2}}{(h+1)(h+2)} - \frac{1}{6} r T_2^3 + \frac{\beta T_2^{g+2}}{h+2}\right)$$

3. Shortage occurs during period T_4 is,

$$BA = k \sum_{i=1}^{n} C_{ib}^{i} \int_{0}^{T_4} I_{r3}(T_4)\, e^{-r(T_3+t)} dt$$
$$= k \sum_{i=1}^{n} C_{ib}^{i} \left(-B \sum_{k=1}^{v} D_{c_{rk}}\right)\left[(1 - rT_3)T_4^2 - \frac{rT_4^3}{2}\right]$$

4. Lost sales occurs during period T_4 is indicated by the difference between shortages and the partial backlog. Therefore,

$$LS = k \sum_{k=1}^{n} C_{ib}^{i} \int_{0}^{T_4} \left(D_{c_{rk}} - BD_{c_{rk}}\right) e^{-r(T_3+t)} dt$$
$$\approx k \sum_{k=1}^{n} C_{ib}^{i}\, D_{c_{rk}} (1-B)\left((1 - rT_3)T_4 - rT_4^2\right)$$

5. Item cost that includes loss due to deterioration as well as the item sold is,

$$IT_r = k \sum_{i=1}^{n} C_b^{i} \left(D_c\left[T_3 + \frac{1}{g+1} hT_3^{g+1}\right] + BT_4 D_c[1 - r(T_3 + T_4)]\right)$$

Therefore, the total cost of the inventory model of SCM 3-echelon can be formulated as the sum of the total cost of suppliers, manufacturers, and retailers as follows.

minimize $TUC(k, T_2, T_4) = TUC_w + TUC_p + TUC_r$
subject to $T_2 > 0, 0 < T_4 < \frac{T}{k}$,
with $T = T_1 + T_2, T_5 = \frac{T}{k}$, and $T_3 + T_4 = T_5$.

IV. Conclusion

An SCM model of 3-echelon multi-tiers has been developed, and the model is comprised 3 multi-entities, i.e., suppliers, producers, and retails. The model was developed using deterioration rate assumption of raw material and finished product following Weibull distribution rate. The joint decision reducing the total cost of the entire system as compared with the decision that taken independently. The system uses multiple-buyer and multiple-deliveries is the most important policy to reduce the use of inventory as well as with the use of the cost. The value of k (the number of delivery) can optimize the total cost of a particular system. For the future work, it is interesting to investigate the model development that can be used in systems that do not have a fixed rate of requests.

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